A numerical method for imaging of biological microstructures by VHF waves

Guido Ala\textsuperscript{a}, Pietro Cassara\textsuperscript{b}, Elisa Francomano\textsuperscript{c,1}, Salvatore Ganci\textsuperscript{a}, Giuseppe Caruso\textsuperscript{d}, Pio D. Gallo\textsuperscript{e}

\textsuperscript{a}Università degli Studi di Palermo, DEIM, Palermo, Italy
\textsuperscript{b}CNR, Istituto di Scienza e Tecnologie dell’Informazione, Pisa, Italy
\textsuperscript{c}Università degli Studi di Palermo, DICGIM, Palermo, Italy
\textsuperscript{d}Università degli Studi di Palermo, DIBIMEF, Palermo, Italy
\textsuperscript{e}Università degli Studi di Palermo, DICHIRONS, Palermo, Italy

Abstract

Imaging techniques give a fundamental support to medical diagnostics during the pathology discovery as well as for the characterization of bio-medical structures. The imaging methods involve electromagnetic waves in a frequency range that spans from some Hz to GHz and over. Most of these methods involves ionizing waves and scanning of wide human body area even if only a focused inspection is needed. In this paper, a numerical method to evaluate the shape of microstructures for application in the medical field, with a very low invasiveness for the human body, is proposed. In particular, the tooth’s root canal is considered. In fact, this is one of the hot topic in the endodontic procedures where rotary instruments are widely used. These instruments are subjected to sudden mechanical damage during the surgical process, due to cyclic fatigue directly related to the canal geometrical characteristics. In order to develop improved endodontic procedure so that instrument breakage probability and canal milling precision are optimized, preliminary canal root reconstruction techniques have to be implemented. These techniques are usually based on invasive X-ray imaging. Thus, a low invasive, easy to use imaging technique that can be applied many times on the patient is of great interest. To this aim, a method based on a flexible thin-wire antenna radiating non ionizing VHF waves, is proposed. By measuring the spatial magnetic field distribution in the neighboring, it is possible

\textsuperscript{1}Corresponding author: Elisa Francomano, elisa.francomano@unipa.it

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to reconstruct the microstructures image by estimating the shape of the antenna against a sensors panel. The mathematical model is strictly non-linear and the inverse problem described above is solved numerically; first simulation results are presented in order to show the validity and the robustness of the proposed approach.

Keywords: Method of Moments, non-linear model, inverse problem, Levenberg-Marquardt method, biological microstructures, non ionizing waves

2000 MSC: code

1. Introduction

The support given by imaging techniques to medical diagnostics is fundamental during the pathology discovery as well as for characterization of biological structures [9, 6, 12, 13, 14, 23, 22]. The imaging methods involve electromagnetic waves in a frequency range that spans from some Hz to GHz and over. Hence, the understanding of biological structures response to the electromagnetic field is fundamental. The investigation of the interaction between healthy or pathological biological tissues and electromagnetic waves is still a hot topic [11]. In fact, understanding how an electromagnetic wave interacts with the human body becomes increasingly important if one considers that the most of imaging methods involve scanning of wide areas of the human body, even if only a focused inspection is needed. In this way, although a wide scanning allows to acquire a big amount of data by single exposition, also areas of the body not interested in the diagnostics are exposed to the waves, so increasing the invasiveness for the patient. For this reason, new imaging systems able to analyze only a focused area (confined scanning), are becoming more and more investigated [14]. A big input to micro-imaging systems has been given by the micro-electronic technology, which allows the development of systems for scanning of confined small areas. In fact, emitters and receivers (sensors) can be made smaller and smaller, so that the final size of the imaging systems as well as the wave spots become very small. Generally, the acquired data need to be appropriately elaborated extracting the imaging information by means of inverse optimization algorithms [12, 13, 22]. Through these algorithms high quality information can be extracted from the data acquired by the sensors, even if the quality of the sensors signal is low. In this paper a method for shape recognition of
microstructures with application in medical field is proposed. Precisely, the
tooth’s root canal reconstruction is considered. In fact, this is one of the
hot topic in the endodontic field where rotary instruments are widely used.
These instruments, named files, used for surgical procedure, are subjected to
various types of mechanical stress, directly related to canal characteristics.
In particular, files separation caused by cyclic fatigue is still an unsolved
problem. Moreover, it occurs without warning since no indication of plastic
deformation appears during the surgical process. Cyclic fatigue studies, often
supported by files builders, have shown the influence of canal shape (i.e. an-
gle, curvature radius, length, possible bifurcations) on mechanical stresses, so
deeply modifying the instrument life and, consequently, its breakage [20, 21].
In order to develop improved endodontic procedure trough which breakage
probability of the files and milling precision of the canal are optimized, root
canal reconstruction techniques have to be implemented. These techniques
are usually based on invasive X-ray imaging: in fact, for most of the endodon-
tic procedures this imaging technique has to be performed many times, so
exposing the patient to unwanted X-ray dose amount. Moreover, the effective
canal shape reconstruction remain a critical task [20, 21].

Hence, an imaging method easy to use, with low invasiveness, with a
satisfactory precision, and that can be applied many times on the patient,
can become a fundamental task for these applications.

The proposed method works with non ionizing low power electromag-
netic waves, in the Very High Frequency (VHF) range, in order to achieve a
low invasiveness for the human body. The method uses a system endowed
with a microtransmitter, a sensors panel to acquire the spatial distribution
of the electromagnetic field in the close proximity and an elaboration logic
to acquire and elaborate the sensors signals. The micro-transmitter radiates
the VHF waves by means of a flexible thin-wire microantenna, inserted in
the root canal. Hence, the micro-antenna can be assumed to have the same
shape of the analyzed structure. By measuring the spatial distribution of the
magnetic field in the neighborhood of the thin-wire antenna, it is possible
to reconstruct the microstructure image. In fact, the shape reconstruction
is possible by estimating the location of thin-wire antenna against the sen-
sors panel. The recognition problem of the thin-wire antenna location by
magnetic field measurements can be addressed as an inverse problem. The
thin-wire antenna is supposed to be a sequence of linear segments: given
a model for the characterization of the magnetic field at a set of points in
space (forward problem) and given a set of measurements at these points,
it is possible to solve the inverse problem in terms of the distances of the antenna axis points from the sensor panel. The magnetic field component is selected since the physical domain can be characterized with about uniform magnetic permeability in the VHF range. In this paper, first results about the previous idea are reported concerning two simulated scenarios. Namely, the spatial distribution of the emitted magnetic field is simulated through a numerical model based on the method of moments (MoM), where the first kind Fredholm integral equation is solved by point matching procedure [1, 7, 8, 3, 4, 5, 2]. The field values are used as the measured input for the inverse problem. The Levenberg-Marquart algorithm [18, 15] is used in solving the inverse problem by means of minimization of the Euclidean distance between the measured field and the field generated by a given configuration of thin-wire piecewise antenna. The numerical procedure involved by the algorithm rounds the entries of the Hessian matrix, step by step, and the location of the antenna segments can be estimated with satisfactory precision, so obtaining the image of the canal shape.

The paper is structured as follows. In section 2 the mathematical framework about the proposed method is presented. In section 3 numerical results about reconstruction examples are discussed, then a conclusion about results and ongoing work complete the paper.

2. Numerical approach

The biological thin microstructures taken into account in this paper are shown in figure 1. The Figure shows the typical shape of a root canal: in this case the canal reveals two curvature changes along the whole length. As already underlined, due to a not accurate prediction of these curvature changes, the endodontic files suffer of mechanical stress that can cause unwanted breakage and added problems in the patient. The shape of these microstructures can be estimated by means of an appropriate embedded emitting antenna and by measuring the spatial distribution of the appropriate radiated electromagnetic field component. In particular, the magnetic field may be the best choice together with the selection of the appropriate work frequency range. The proposed approach is a typical inverse problem: the measured spatial distribution of the magnetic field emitted by a VHF thin-wire antenna (i.e. with radius less than one hundredth of the maximum work wavelength) is the input, and the source shape reconstruction is the final target. The antenna is assumed to be a piecewise linear structure, i.e.
Figure 1: Tooth’s root canal (point red line)

a sequence of $N_b$ branches with uniform radius $r_c$ and conductivity $\sigma_c$. The antenna is fed at the point $P_0(x_0, y_0, z_0)$ by a sinusoidal current source with known frequency $f$ and amplitude $I_{\text{source}}$. The spatial distribution of the magnetic field is detected by a sensors panel with $M$ sensors distributed on its surface. As a first approximation, the surrounding medium is supposed to be homogeneous, isotropic, with conductivity $\sigma$, electric permittivity $\epsilon$ and magnetic permeability $\mu$. Under these assumptions, the shape reconstruction involves the minimization of an objective function shown in equation 1, by using an iterative procedure. Roughly speaking, step by step the iterative procedure looks for a set of segments coordinates that minimize the 2-norm difference between the computed and measured magnetic field at sensor points, i.e. the error function $F$:

$$
\min_p \{ F \} = \min_p \{ \| H_{\text{comp}}(p) - H_{\text{meas}}(p) \|_2 \} \tag{1}
$$

In equation (1), $p = [(x_1, y_1, z_1)\ldots(x_{N_b}, y_{N_b}, z_{N_b})]^T$ is the vector of the Cartesian coordinates of the antenna segments ends to be estimated, $H_{\text{comp}}$ is the vector of the magnetic field values at sensors points computed by assuming that the segments ends are located at $p$, and $H_{\text{meas}}$ is the vector with the measured magnetic field values. The values of $H_{\text{comp}}$ are calculated by the forward solver. For a given signal source and antenna characteristics. More precisely, the current distribution along the antenna is evaluated by solving an appropriate integral equation in frequency domain, derived from Maxwell equations. Then, by using the relation that links the currents to the mag-
netic field, this latter can be calculated at the sensors points. The equations of the forward model are solved numerically. For the problem addressed in this case, the MoM via point-matching procedure in the frequency domain, is performed. Note that, by this method the involved integral equation includes the boundary conditions, so that only the emitter have to be discretized but not the whole problem domain. This means that each antenna branch needs to be split into a finite number of linear segments [1, 7, 8, 3, 4, 5]. The problem formulation leads to a first kind Fredholm integral equation. In fact, by expressing the electric field $\vec{E}$ with the retarded magnetic vector potential $\vec{A}$, the source frequency ($\omega = 2\pi f$) and the scalar electric potential $\phi$, as shown in equation 2:

$$\vec{E} = -(\nabla \phi + j\omega \vec{A})$$

(2)

and by introducing the boundary conditions by means of the per-unit-length surface impedance $\dot{z}_s$, the following equation holds:

$$\vec{u} \cdot \vec{E} = \dot{z}_s I_s$$

(3)

$-(\nabla \phi + j\omega \vec{A})_{tg} = z_s I_s$

in which the subscript $tg$ indicates the tangential component to the wire surface, $I_s$ is the longitudinal current flowing into the conductor, supposed concentrated on its axis ($u'$ as the unit vector) because of the thin-wire assumption [1, 3], $u$ is the unit vector tangential to the conductor’s surface, as shown in figure 2.

![General thin-wire geometric references.](image)

Figure 2: General thin-wire geometric references.

By using the relation between the magnetic vector potential $\vec{A}$ and the conduction current density, and the relation between the scalar electric potential $\phi$ and the free electric charge density, the well known electric field
integral equation (EFIE) is obtained. Then, by integrating the EFIE along the surface of the conductor, the following modified EFIE is obtained \[1, 3\]:

\[
j \omega \int_L \bar{\mathbf{u}} \cdot \int_{\Omega} \mu I_s(l') \hat{g}(\bar{r}, r') dl' dl + \int_L \frac{\partial}{\partial x} \frac{1}{j \omega \epsilon} \int_{\Omega} \frac{dI_s(l')}{dl'} \hat{g}(\bar{r}, r') dl' dl = -\hat{z}_s \int_L I_s(\bar{r}) dl
\]

Equation (4) is a general relation that depends only on the longitudinal current and on geometrical quantities related to the conductors constituting the thin-wire structures to be analyzed. In fact, \(\Omega\) is the length of the exciting conductor, \(L\) is the length of the induced conductor, \(\bar{r}\) and \(r'\) are space position vectors of the observation and source points, respectively; \(g(\bar{r}, r') = \frac{e^{-jkr'}}{4\pi|\bar{r} - r'|}\) is the Green’s function in an unbounded region. The quantity \(\hat{\epsilon} = \epsilon + j \omega \sigma\) takes into account the complex medium permittivity and \(\hat{k} = \sqrt{-\omega^2 \mu \hat{\epsilon}}\) is the wave number. As already underlined, the forward problem, represented by equation (4), is numerically solved by splitting the thin-wire antenna into a finite number of linear segments of length \(\Delta\). In scientific literature numerous results \[1, 3\] show that an acceptable accuracy can be obtained for the solution of equation (4), by assuming \(\Delta \leq 0.05 \lambda\) and a spatial linear distribution of current along the axis of each segment. In this way, a linear system of order \(n + N_b\) is obtained with \(n\) as the number of segments. Once the currents are computed, the magnetic field components in the surrounding medium are given by the dipole theory, by superposing the effects of all segments \[1, 3\]. As discussed above, once the direct problem is solved, the inverse one can be then approached. In Figure 3 a block diagram for the algorithm used to solve the inverse problem is shown. The unknown end-points segments coordinates \(s\) are obtained through an iterative procedure which, at each step, computes the correction factors needed to obtain the new set of points coordinates. The correction factors are evaluated by the optimization functionality, on the basis of the difference in 2-norm among the measured fields and the fields computed by the forward solver. Since the problem is strictly nonlinear, the inverse problem solver is based on the Levenberg-Marquardt (LM) algorithm \[18\]. In brief, the LM algorithm starts from an initial set of reasonable end-points segments coordinates \(s^{(0)}\); this set is then updated at each step \(k\) by adding the solutions \(\Delta s^{(k)}\) of the
linear system shown in equation (5):

\[
\left( \tilde{B}^{(k)} + \beta^{(k)} \text{diag} [\tilde{B}^{(k)}] \right) \Delta s^{(k)} = - G^{(k)^T} F(s^{(k)})
\]

(5)

where the matrix \( \tilde{B}^{(k)} \) is an approximation of the Hessian matrix of \( F \) at step \( k \) and is equal to \( G^{(k)^T} G^{(k)} \), with \( G^{(k)} \) as the gradient of the error function \( F \) at step \( k \), while \( \beta^{(k)} \) is an adaptive parameter. Once \( \Delta s^{(k)} \) is known, the new coordinates are calculated by the following relation:

\[
s^{(k+1)} = s^{(k)} + \Delta s^{(k)}
\]

(6)

3. Numerical results

In order to validate the capabilities of the proposed method, some numerical experiments concerning the shape estimation of a specific tooth’s root canal have been carried out.

A NiTi (electrical conductivity 1.1 \( \cdot \) 10^6 S/m) thin-wire antenna 0.1 mm in radius and 2.3 cm in length is assumed to be inserted in the root canal and fed by a sinusoidal current source with frequency equal to 100 MHz. The NiTi alloy is selected because it is commonly used in biomedical applications [20, 21]. The antenna shape, which approximate the canal shape, is shown in Figure 4a and Figure 4b (red dashed line with cross markers). The same figures show also that the ground surface of the antenna lies on the \( Y-Z \)
plane, and the feed point is placed at a distance of 2 mm from the ground plane. The sensors are assumed to be sensitive to the component of the magnetic field along the $Z$ axis.

In order to approximate the characteristics of a tooth, the medium around the canal is assumed to have zero electrical conductivity, relative permittivity equal to 15 and relative permeability equal to $1 \ [16]$. Two sets of simulations have been carried out: one set concerning a noiseless environment, and one set to test the influence of noise on the shape reconstruction.

In the noiseless context, the Signal to Noise Ratio (SNR) is assumed to approach infinite: the measured values of magnetic field have been calculated by means of the MoM-based forward solver described in section 2. Different current source amplitudes have been tested: 0.1 mA, 1 mA, 5 mA, 25 mA and 50 mA. It has to be underlined that all these current values as well as the selected frequency, can be well tolerated by the human tissues also depending on the time of application (i.e. a few seconds) [10, 11, 17, 19]. On the other hand, the VHF waves are non ionizing ones with respect to the commonly used X-ray.

Two sets of 12 sensors are considered. The sensors are placed on a flat surface in a first case study (Figure 4a) and on a semi-cylindrical surface in a second case study (Figure 4b). The flat sensor panel is 3 cm in height and 1.5 cm in width, while the semi-cylindrical one is 1.5 cm in radius and 3 cm in height.

The antenna shape has been reconstructed by means of a three branch conductor. The initial set of end-points segments coordinates $s^{(0)}$ for the LM optimization algorithm is that of a straight thin-wire antenna 2.3 cm in length and perpendicular to the ground plane. From these coordinates the true position of each branch is then estimated.

The estimated conductor position in the flat sensors panel case is shown in Figure 4a. In this first noiseless case study, the distance between the estimated shape (black dashed line with square markers) and the true position (the red dashed line with cross markers) is close to zero.

For this configuration, the $2$-norm relative error between true and estimated coordinates values for a given current level, and the number of iterations are reported in Table 1. The results show that the error is close to zero, also for small values of the source current.

Figure 4b shows the estimated position of the conductor for the semi-cylindrical sensor panel. For this scenario, the $2$-norm relative error between
Figure 4: Noiseless shape reconstructions. The initial position is that of a straight thin-wire antenna 2.3 cm in length and perpendicular to the $Y - Z$ plane.
Table 1: Relative error and number of iteration required with the flat sensors panel

<table>
<thead>
<tr>
<th>Current [mA]</th>
<th>Relative error</th>
<th>No. of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>6.15e-11</td>
<td>169</td>
</tr>
<tr>
<td>1</td>
<td>6.10e-11</td>
<td>165</td>
</tr>
<tr>
<td>5</td>
<td>7.88e-09</td>
<td>87</td>
</tr>
<tr>
<td>25</td>
<td>3.78e-10</td>
<td>28</td>
</tr>
<tr>
<td>50</td>
<td>1.53e-11</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2: Relative error and number of iteration required with the semi-cylindrical sensor panel

<table>
<thead>
<tr>
<th>Current [mA]</th>
<th>Relative error</th>
<th>No. of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.05e-10</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>1.04e-09</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>1.75e-11</td>
<td>12</td>
</tr>
<tr>
<td>25</td>
<td>2.55e-12</td>
<td>8</td>
</tr>
<tr>
<td>50</td>
<td>1.00e-09</td>
<td>8</td>
</tr>
</tbody>
</table>

true and estimated coordinates for a given level of source current, and the number of iterations are shown in Table 2. The simulations results show that the error is close to zero also in this case.

The second set of experiments is aimed to study the influence of noise on reconstruction accuracy.

The measurements are supposed to be affected by additive white Gaussian noise: the measured values of magnetic field have been calculated by means of the MoM-based forward solver described in section 2 and then the noise has been added.

Our experiments showed that the sensor panels tested in the ideal noiseless case are not able to provide an acceptable reconstruction accuracy in a noisy environment. Therefore more complex sensor panels has been tested, namely 24 or 32 sensors on two parallel flat panels (each panel is 3 cm in height and 1.5 cm in width, see Figure 6 and Figure 7 respectively) and
32 sensors on a cylindrical panel (1.5 cm in radius and 3 cm in height, see Figure 8).

Moreover, in order to test the effect of an increasing number of variables involved in the optimization process on reconstruction accuracy, the shape of the antenna has been reconstructed by assuming a two branch conductor or a three branch conductor, alternatively: if one considers common canal shapes and curvatures, both of these configurations can potentially represent the canal shape in a satisfactory way.

The behavior of the relative reconstruction error versus the SNR for different sensor panels and numbers of assumed antenna branches is shown in Figure 5. In Figures 6-8, some reconstructions are also shown for low values of SNR. In all these experiments, the current source amplitude has been set to 50 mA.

![Figure 5: Relative reconstruction error vs. SNR (dB).](image)

It turns out that satisfactory accuracies can be achieved with all the configurations tested, even when the SNR is low. However, by approximating the antenna with a two branch conductor, a better robustness with respect
to noise can be achieved, without compromising accuracy. In particular, two branch conductors reconstructions outperform three branch ones for very noisy environments. In general, for a fixed number of conductor branches, the best results are obtained by employing flat sensor panels.

4. Conclusions

In this paper, a numerical approach to reconstruct the shape of microstructures for bio-medical application with a very low invasiveness for the human body, is proposed. The tooth’s root canal reconstruction is considered. This is one of the hot topic in the endodontic procedures where there is a widely use of rotary instruments (files) subjected to various types of mechanical stress, directly related to the canal characteristics. Files separation, caused by cyclic fatigue, occurs without warning so determining unwanted failure of the endodontic procedure also with potential surgery complication for the patient. Therefore, preliminary root canal reconstruction techniques have to be implemented in order to optimize the endodontic procedure. These techniques are usually based on invasive X-ray imaging. In fact, for most of the endodontic procedures this imaging technique has to be performed many times, so exposing the patient to unwanted X-ray dose amount. Moreover, the effective canal shape reconstruction remain a critical task. In the paper, a low invasive, easy to use, imaging technique that can be applied many times on the patient, is proposed. It is based on a flexible thin-wire antenna radiating non ionizing VHF waves and inserted in the canal. The typical inverse problem is solved numerically: the estimation of the antenna shape enables to reconstruct the canal image with a satisfactory approximation. First simulation results also show the validity and the robustness of the proposed approach. A prototype of the actual application is currently under construction.

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References

Figure 6: Shape reconstruction in noisy environment, two flat sensor panels (24 sensors)
Figure 7: Shape reconstruction in noisy environment, two flat sensor panels (32 sensors)
Figure 8: Shape reconstruction in noisy environment, cylindrical sensor panel (32 sensors)

(a) Two segments.

(b) Three segments.


